Problem Set 1

Due date: October 11, in class

Exercise 1. *Let* $a < b$*. Define* $A_n = [a + \frac{1}{n}, b - \frac{1}{n}]$ *and* $B_n = (a - \frac{1}{n}, b + \frac{1}{n})$*, where* $n \in \mathbb{N}$*. Find* $\cup_{n=1}^{\infty} A_n$ *and* $\cap_{n=1}^{\infty} B_n$*.*

Exercise 2. *Given a function* $f : X \to Y$ *and its inverse function* f^{-1}*. Let* $B \subset Y$*. Show that* $f(f^{-1}(B)) \subset B$*. Under what additional conditions do we have* $f(f^{-1}(B)) = B$*?*

A binary relation R on X is *complete* if for all $x, y \in X$, xRy or yRx. R is *rational* if it is complete and transitive. Function $u : X \to \mathbb{R}$ is a *utility function representing* R if for all $x, y \in X$, xRy iff $u(x) \geq u(y)$.

Exercise 3. *Show that* R *can be represented by a utility function only if it is rational.*

Exercise 4. *Let* $f : \mathbb{R} \to \mathbb{R}$ *be a strictly increasing function. Show that if* R *can be represented by* u*, it can also be represented by the composite function* $f \circ u$*.*